SERIES OF DIFFERENCE TONES OBTAINED FROM TUNABLE BARS[1]

By Paul Thomas Young

Several years ago we noticed that unusually clear and loud difference tones could be obtained by striking simultaneously two bars in the upper octave of a standard set of orchestra bells. Although difference tones may be produced throughout the entire range of the instrument, which is two and a half octaves, they are especially distinct in the region from e^4 to g^4. The top octave, c^4 to c^5, contains frequencies ranging approximately from 2000 vd. to 4000 vd.[2]

Bars make an admirable type of apparatus for group demonstrations since they are easy to manipulate, simple in construction, and further since they make it possible to produce series, musical sequences and clangs of difference tones. The resultant tones are often more easily apprehended than the generating tones themselves. Demonstrational series of difference tones, obtained from a standard set of orchestra bells, were readily heard by groups of 350 elementary students in the University of Illinois.

In the following account we shall describe (1) series of difference tones obtained from a standard set of orchestra bells, and (2) tunable bars arranged for experimental study and for the demonstration of difference tones.

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[1]From the Psychological Laboratory, University of Illinois.

[2]It is interesting to note that Ellis, translating Helmholtz, uses tones in this region for the demonstration of difference tones. He writes: "I have found that combinational tones can be made quite audible to a hundred people at once, by means of two flageolet fifes or whistles, blown as strongly as possible. I choose very close dissonant intervals because the great depth of the low tone is much more striking, being very far below anything that can be touched by the instrument itself. Thus g'''' being loudly sounded on one fife by an assistant, I give $f''''\sharp$, when a deep tone is instantly heard which, if the interval were pure, would be g, and is sufficiently near to g to be recognized as extremely deep. As a second experiment the g'''' being held as before, I give first $f''''\sharp$ and then e'''' in succession. If the intervals were pure the combinational tones would jump from g to c'', and in reality, the jump is very nearly the same and quite appreciable." Helmholtz, H. L. F., *On the Sensations of Tone*, etc., Ellis trans., 1895, 153.

I. Difference Tones Obtained from Orchestra Bells. Table I shows the approximate pitch of difference tones obtained by combining pairs of bars in the top octave, c^4 to c^5, of the instrument. The pitch of the difference tones has been estimated on the assumption that the intervals are just, while, as a fact, they are intervals of an equally tempered scale. This means that the actual pitches will in some cases be higher and in other cases lower than the values indicated in the table.

TABLE I*

	$c\#^4$	d^4	$d\#^4$	e^4	f^4	$f\#^4$	g^4	$g\#^4$	a^4	$a\#^4$	b^4	c^5
c^4	$c\#$	c^1	$g\#^1$	c^2	f^2	$g\#+^2$	c^3					
$c\#^4$		d	$c\#^1$	a^1	$c\#^2$	$f\#^2$	$a+^2$	$c\#^3$				
d^4			$d\#$	d^1	$a\#^1$	d^2	g^2	$a\#+^2$	d^3			
$d\#^4$				e	$d\#^1$	b^1	$d\#^2$	$g\#^2$	$b+^2$	$d\#^3$		
e^4					f	e^1	c^2	e^2	a^2	$c+^3$	e^3	
f^4						$f\#$	f^1	$c\#^2$	f^2	$a\#^2$	$c\#+^3$	f^3
$f\#^4$							g	$f\#^1$	d^2	$f\#^2$	b^2	$d+^3$
g^4								$g\#$	g^1	$d\#^2$	g^2	c^3
$g\#^4$									a	$g\#^1$	e^2	$g\#^2$
a^4										$a\#$	a^1	f^2
$a\#^4$											b	$a\#^1$
b^4												c^1

*Difference tones produced by minor and major sixths have been omitted from the table, since these intervals were found to generate simultaneous first and second order difference tones. This corresponds to the findings of Krueger, Meyer, and others (references at close of this paper). For purposes of preliminary demonstration it seemed best not to complicate matters through the introduction of higher order difference tones.

We might add that f^2 combined with $f\#^2$, and also $f\#^2$ combined with g^2, which are the lowest bars upon our instrument, yield unusually clear summation tones. The tone of the first combination can readily be heard as higher than that of the second.

Difference tones exaggerate the defects of equal temperament. Since the pitch of the difference tone equals the absolute vibration difference between the generators, a small change of one of the generators, say n vibrations, makes a greater relative change in the difference tone.[3]

[3]The interval c^4—$c\#^4$, for example, does not correspond to the just ratio 15/16 but to a ratio slightly short of this, 84/89. If the ratio were just, the difference tone would lie four octaves below the $c\#^4$ generato.r It is found to be flat. In general, the semitone, tone, minor third, and fifth are too short in an equally tempered scale: the major third, fourth, and tritone are too long. (Calculated, *e.g.* from data in Helmholtz, *op. cit.*, Appendix, 453-456.)

We list below some of the series of difference tones which are useful for demonstrational purposes.

(1) By keeping a constant interval between the generators —as a semitone, a tone, a minor third, a major third, a fourth, a tritone, or a fifth— and by varying the generators themselves up and down the register, musical scales and simple melodies can be produced in the difference tones. If, while playing a melody, one attempts to change the interval between the generators, the disturbances which are due to temperament are at once apparent; but so long as one works with a constant interval, the difference tones have the correct tempered ratios among themselves (provided the instrument be in tune).

Chromatic series of difference tones, produced by generators which remain in a constant ratio, are to be found along the diagonals of Table I. For example, the following series of generators gives a chromatic series of difference tones:

Generator 1	$c\sharp^4$	d^4	$d\sharp^4$	e^4	f^4	$f\sharp^4$	g^4	$g\sharp^4$	a^4	$a\sharp^4$	b^4	c^5
Generator 2	c^4	$c\sharp^4$	d^4	$d\sharp^4$	e^4	f^4	$f\sharp^4$	g^4	$g\sharp^4$	a^4	$a\sharp^4$	b^4

By running down the instrument in semitones the series of difference tones becomes constantly lower and fainter and is nearly inaudible in the region of c^3. The loudest difference tones are obtained near the region from e^4 to g^4. This region, according to Helmholtz,[4] contains tones which are reënforced by the natural resonance of the ear. Generating tones above this critical region do not produce as loud difference tones as those within the region or near it. It is a fact of considerable theoretical importance that the loudest difference tones are produced from generators which receive natural resonance from the ear. We hope that further observations will be made upon this point.

(2) A descending series of difference tones may be produced from an ascending series of generators, or *vice versa*. The condition underlying such a series is that the absolute vibration difference decrease (or increase) as the generators ascend (or descend). The following series produces descending difference tones from ascending generators or, when played in the reverse order, ascending difference tones from descending generators:

Generator 1	a^4	$a\sharp^4$	b^4	c^5
Generator 2	f^4	g^4	a^4	b^4

[4] Helmholtz, *op. cit.*, 116, 179.

(3) A constant upper generator combined with a descending lower generator gives an ascending series of difference tones; played in the reverse order, a descending series. Following is an example:

Generator 1	c^5	c^5	c^5	c^5	c^5	c^5	c^5
Generator 2	b^4	$a\sharp^4$	a^4	$g\sharp^4$	g^4	$f\sharp^4$	f^4

A constant lower generator combined with an ascending upper generator gives an ascending series of difference tones; played in the reverse order, a descending series. Following is an example:

Generator 1	$f\sharp^4$	g^4	$g\sharp^4$	a^4	$a\sharp^4$	b^4	c^5
Generator 2	f^4	f^4	f^4	f^4	f^4	f^4	f^4

Series of difference tones of approximately constant pitch (which would be of identical pitch were the intervals just) may be obtained from ascending or descending series of generators. Reference to Table I shows that the same difference tone is produced by the following combinations of generating tones:

Generator 1	f^4	a^4	c^5
Generator 2	c^4	f^4	a^4

(4) Leaps of approximately an octave (based, not upon Table I, but upon actual tests with equally tempered intervals) may be produced by the following sequences:

Generator 1	$f\sharp^4$	g^4	$g\sharp^4$	a^4	$a\sharp^4$	b^4
Generator 2	f^4	f^4	g^4	g^4	a^4	a^4

Tones approximately in the ratio 4:5:6 may be obtained from the following generators:

Generator 1	f^4	$f\sharp^4$	g^4	g^4	$g\sharp^4$	a^4	a^4	$a\sharp^4$	b^4
Generator 2	c^4	c^4	c^4	d^4	d^4	d^4	e^4	e^4	e^4

II. *Tunable Bars for Experimental Study, and for the Demonstration of Difference Tones.* In order to make an apparatus capable of delicate adjustment and accurate control, and in order to avoid the difficulties arising from the use of equally tempered intervals—difficulties which are inevitable with orchestra bells and which are magnified in the difference tones

—we have made use of the tunable bar, previously described.[5]

Round top steel bars were obtained[6] and slotted from one end to the nodal line, which is approximately .22 of the length of the bar from each end. In this slot we placed a bolt (1/16 inch diameter, ¾ inch length) which carries a lock-nut. The bolt may be moved along the slot and clamped at any point. This arrangement makes accurate tuning possible.

We found that, by striking a bar on the rounded edge, two vibratory components were produced and also their first differ-

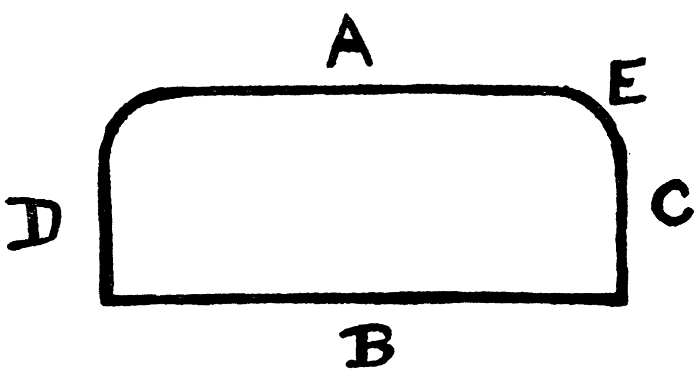

FIGURE I

ence-tone. Figure I represents the cross-section of a bar in the center of its length. If the hammer strike at a point *A*, a tone is produced whose frequency varies directly as the thickness *AB*, and a blow at *C* produces a tone whose frequency

[5]Young, P. T., Tunable Bars, and Some Demonstrations with a Simple Bar and a Stethoscope. *Psychol. Bull.*, 1918, 15, 293 f.

Dr. C. R. Griffith has called our attention to the fact that the tone of vibrating bars is intensified by bringing a metal or cardboard funnel over the center. Although this funnel may act in part as a resonator, its chief function is probably to reduce the interference of the opposite-phase trains of sound waves which arise simultaneously from the center and ends of the bar. Intensification by means of a funnel is simpler than intensification with a stethoscope, the method which we have previously used. We might add that tunable bars have proved satisfactory for the study of beats in the elementary laboratory.

[6]The bars were purchased, at $1.00 a bar, from J. C. Deagan Co., Ravenswood and Berteau Aves., Chicago. The heavy metal hammers, necessary for striking the bars, cost 60 cents.

The bars have a cross-section of 1.1 x 2.6 cm. and, after tuning, the following lengths, in cm.:

Frequency of Bar	2600	2700	2800	3000	3200	3400	3600	3800
Length	14.1	13.5	13.4	13.0	12.6	12.3	11.9	11.6

varies directly as the thickness *CD*. Now if the blow of the hammer fall on the rounded edge *E*, two vibratory components are present and a difference tone is heard. By varying the cross-section of a bar it is possible to vary the ratio of these component tones, and hence to control the difference tone arising from them. But for our purposes, in order to avoid effects due to the lateral vibratory component, the bars have been mounted upside down, so that a flat surface is presented to the hammer.

The nine bars in our series are supported upon tightly stretched cords which pass through screw-eyes. The screw-eyes are arranged in two lines upon the base of a wooden frame and set at such distances apart that each bar is supported immediately beneath its nodal lines.

TABLE II

	2600	2700	2800	3000	3200	3400	3600	3800
Variable bar 2500 to 2600	0–100	100–200	200–300	400–500	600–700	800–900	1000–1100	1200–1300
2600		100	200	400	600	800	1000	1200
2700			100	300	500	700	900	1100
2800				200	400	600	800	1000
3000					200	400	600	800
3200						200	400	600
3400							200	400
3600								200

Table II shows all possible difference tones obtainable from our series of bars. The steps between bars of 100 and 200 vd. were selected because the laboratory possesses a set of forks (made by Spindler and Hoyer, Göttingen) which are tuned to the frequencies: 100, 200, 300, 400, 500, 600, 700, 800. For demonstrational purposes it is convenient to compare fork tones and difference tones.[7]

At the start we planned to tune the bars to the ratios 8:9:10:11:12:13:14:15:16. We soon found, however, that when the interval exceeded a fifth—when, for example, we worked with the minor or major sixth—two simultaneous difference tones were heard.[8] Consequently, for our purposes, it seemed best to

[7]We first tuned the 3200 bar to the double octave of the 800 fork and then the 2800 bar to the double octave of the 700 fork. Having established the frequencies of these two bars, and checked the difference tone with the 400 fork, we tuned the other bars entirely upon the basis of their difference tones. See Stumpf, C., Ueber die Bestimmung hoher Schwingungszahlen durch Differenztöne, *Ann. d. Physik u. Chemie* (Wiedermann's), N. F., 1899, LXVIII, 105-116; Appunn, A., Warum können Differenztöne nicht mit Sicherheit zur Bestimmung hoher Schwingungszahlen angewandt werden? *ibid.*, 1899, LXVII, 222–226.

[8]For discussions of the higher order difference tones see the papers by Krueger, Meyer, and others, references to which are appended.

limit our frequencies to those within the ratio of a fifth, 2:3 (we have actually stepped over this limit a bit). The fifth selected [illegible], in the region where difference tones are loudest, *i.e.*, in the neighborhood of e^4 to g^4.

The following demonstrations may be readily made with the tunable bars.

(1) Difference tones of the same pitch may be produced from generators of different frequencies. If, for example, we strike any two adjacent bars, disregarding the variable and the 2700 bar, we hear a difference tone of 200 vd. Similarly difference tones of 400, 600, 800, and 1000 can be produced by several different combinations of bars (Table II). The 2700 bar makes it possible to obtain a difference tone of 100 vd. at two places, and also it gives the series: 100, 300, 500, 700, 900 and 1100.

(2) The lowest bar of the series is provided with a thumbnut and a load which is somewhat greater than that of the other bars, so that it may be varied continuously from 2600 vd. down to 2500. If combined with the 2600 bar, it yields a continuous series of difference tones from 0 to 100 vd. If the variable bar is combined successively with the 2600, 2700 and 2800 bars, a continuous series of difference tones from 0 to 300 vd. can be produced. This type of apparatus is admirable for studying the lower limit of difference tones and for other similar problems. It would be a comparatively simple matter to construct a series of tunable bars which would yield a continuous series of difference tones from 0 to 1200 vd. or above.

TABLE III

Ratios	Difference Tones	Generators			
1	50	2550	2600		
2	100	2600	2700		
3	150	2550	2700		
4	200	2600	2800		
5	250	2550	2800		
6	300	2700	3000		
1	100	2700	2800		
2	200	2800	3000		
3	300	2700	3000		
4	400	2800	3200		
5	500	2700	3200		
6	600	2800	3400		
1	200	2600	2800	2600	3800
2	400	2600	3000	2800	3800
3	600	2600	3200	3000	3800
4	800	2600	3400	3200	3800
5	1000	2600	3600	3400	3800
6	1200	2600	3800	3600	3800

(3) Series of difference tones, having the simple ratios 1:2:3:4:5:6 may be readily produced. Table III shows three such series, the first being obtained by setting the variable bar at 2550 vd. If we call a tone of 50 vd. *C*, then the following sequence may be produced by combining the tones shown in Table III:

$C \quad c \quad g \quad c' \quad e' \quad g' \quad c'' \quad e'' \quad g'' \quad c''' \quad e''' \quad g'''$

Here is a musical sequence extending through four and a half octaves produced by nine bars whose frequencies are all included within the interval of a minor sixth.

(4) Musical clangs, made up of difference tones, may be produced by striking simultaneously or in rapid succession bars which give only the small ratio numbers (1, 2, 3, 4, 5, 6). Table

TABLE IV

Clang No. 1 Bars 2600 2800 3000 3200 3400 3600 3800
Difference tones: 200, 6 times; 400, 5 times; 600, 4 times; 800, 3 times; 1000, 2 times; 1200, 1 time.

Clang No. 2 Bars 2600 2700 2800 3000 3200
Difference tones: 100, 2 times; 200, 3 times; 300, 1 time; 400, 2 times; 500, 1 time; 600, 1 time.

Clang No. 3 Bars 2550 2600 2700 2800
Difference tones: 50, 1 time; 100, 2 times; 150, 1 time; 200, 1 time; 250 1 time.

IV shows three such clangs. If one play successively these three clangs in the order 1, 2, 3, a series of musical chords will be heard, each one deeper in pitch and richer than the preceding. It should be remembered that to produce a musical clang of difference tones the generators themselves must stand in such simple numerical ratios as 13:14:15:16:17:18:19 or 26:27:28:30 or 51:52:54:56. We have calculated that with comparatively few bars it is possible to get a complete scale of difference tones in just temperament, and also from the same bars to produce difference tone clangs related as tonic to dominant, or as tonic to subdominant.

If the bars are struck energetically, the clang has a piercing quality which may be almost painful, and which is localized in the ear. When the bars are played lightly, or with a moderate blow of the hammer, the musical chord is rich, full and true while the total clang is very bright, on account of the loudness and high pitch of the generators. Difference tone clangs are agreeable and, we believe, are not without musical value.

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N.B. The C. H. Stoelting Co., 3037-3047 Carroll Ave., Chicago, are manufacturing sets of bars of the sort described in the present paper for the demonstration and experimental study of difference tones. Also they are preparing differential bars for work with beats and the tonal *DL* (see foot-note 5). At the present time no price can be mentioned.